Chapter Seven

Solar Energy

Why Studying Solar energy

- To know the heat gain or heat loss in a building
- In making energy studies
- In the design of solar passive homes.

Thermal Radiation

- Solar spectrum is composed of: 1-Ultraviolate light 2-visible light 3- infrared
- **Thermal radiation**: is the part of electromagnetic spectrum that primarily cause heating effect, which is part of ultraviolet, infrared and visible light. And it has a weave length of 0.1×10^{-6} m and it is mainly expressed in microns
- **Total or global irradiation (G):** is the total thermal radiation impinges on a surface from all directions and from all sources in w/m^2 or $Btu/(hr\text{-}ft^2)$
- **Thermal radiation is divided into:** Absorption, reflection and transmission
- **Absorption:** is the transformation of the radiant energy into thermal energy stored by the molecules.
- **Reflection (reflectance):** is the return of radiation by a surface without change of frequency
- **Transmission (transmittance):** is the passage of radiation through a medium without change of frequency.
- **Radiation:** Energy transfer from a body due to its internal temperature. Hot surfaces radiates energy with shorter wavelength, while colder surfaces radiates energy with longer wavelength according to Wien's displacement law
- **Opaque surface:** a surface with transmittance equals to zero.
- **Black body:** is that body with reflectance equals to zero.

$$\alpha + \rho + \tau = 1 \tag{7-1}$$

where:

α = the *absorptance*, the fraction of the total incident thermal radiation absorbed
ρ = the *reflectance*, the fraction of the total incident thermal radiation reflected
τ = the *transmittance*, the fraction of the total incident radiation transmitted through the body

$$\lambda_{max} = \frac{5215.6}{T} \text{ microns} \tag{7-2}$$

This equation is known as *Wien's displacement law*. The maximum amount of radia-

Earth Motion About The Sun

- **Elcliptic plane or orbital plane:** is the earth orbit around the sun, which takes 365.25 days.
- **The mean distance between earth and sun is 92.9 x 10^6 miles**
- **The perihelion distance:** it happens when the earth is the closest to the sun and it is 98.3% of the mean distance and occurs in January 4.
- **The aphelion distance:** when the earth is farthest from the sun and it is 101.7% of the mean distance and occurs at July 5.
- **Earth rotates around itself with tilted orbit of 23.g deg with respect to the sky**
- **Vernal equinox (الإعتدال الربيعي):** When day time equals night time in March 21.
- **Autumnal equinox (الإعتدال الخريفي):** When day time equals night time in September 22.
- **Summer solstice (ألإنقلاب الصيفي):** which means sun stand still, when the north pole is inclined 23.5 deg towards sun, so the northern hemisphere have summer and southern part have winter. Opposite happens in winter solstice.
- **Torrid zone:** is the region between, where the sun is at zenith (directly overhead) at least once during the year.
- **Frigid zones:** those zones with latitude greater then 66.5 deg, where the sun is below horizon for at least one day each year.

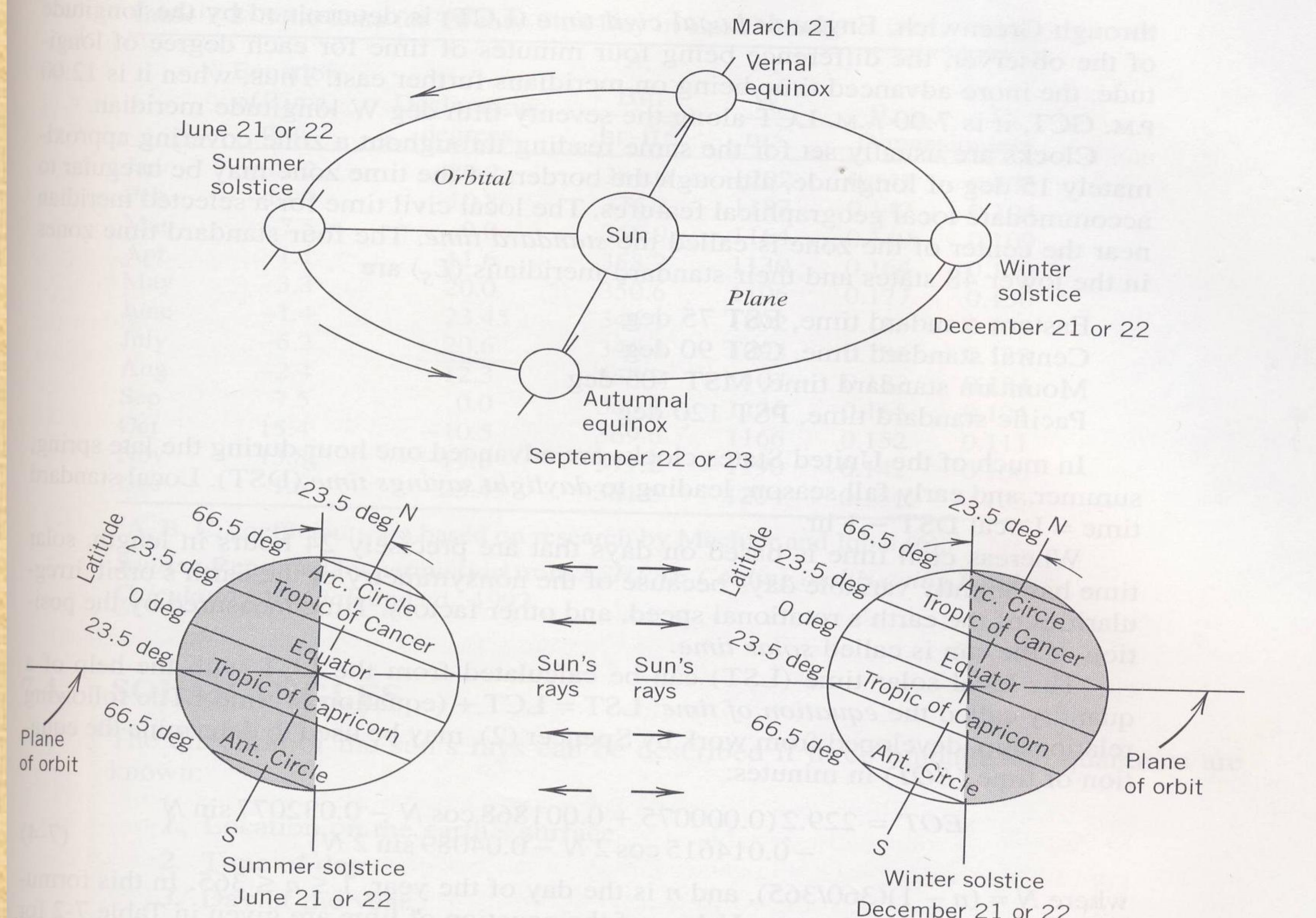

Figure 7-1 The effect of the earth's tilt and rotation about the sun.

Table 7-1 Solar Absorptances

Surface	Absorptance
Brick, red (Purdue) [a]	0.63
Paint, cardinal red [b]	0.63
Paint, matte black [b]	0.94
Paint, sandstone [b]	0.50
Paint, white acrylic [a]	0.26
Sheet metal, galvanized, new [a]	0.65
Sheet metal, galvanized, weathered [a]	0.80
Shingles, aspen gray [b]	0.82
Shingles, autumn brown [b]	0.91
Shingles, onyx black [b]	0.97
Shingles, generic white [b]	0.75
Concrete [a,c]	0.60–0.83
Asphalt [c]	0.90–0.95
Grassland [d]	0.80–0.84
Deciduous forest [d]	0.80–0.85
Coniferous forest [d]	0.85–0.95
Snow, fresh fallen [c]	0.10–0.25
Snow, old [c]	0.30–0.55
Water, incidence angle 30°	0.98
Water, incidence angle 60°	0.94
Water, incidence angle 70°	0.87
Water, incidence angle 85°	0.42

Sources

[a] F. P. Incropera and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer,* 3rd ed., John Wiley & Sons, New York, 1990.

[b] D. S. Parker, J. E. R. McIlvaine, S. F. Barkaszi, D. J. Beal, and M. T. Anello, "Laboratory Testing of the Reflectance Properties of Roofing Material," FSEC-CR670-00, Florida Solar Energy Center, Cocoa, FL.

[c] A. Miller, *Meteorology*, 2nd ed., Charles E. Merrill Publishing, Columbus, OH, 1971.

[d] J. M. Moran, M. D. Morgan, and P. M. Pauley, *Meteorology—The Atmosphere and the Science of Weather*, 5th ed., Prentice Hall, Englewood Cliffs, NJ, 1997.

Time and Solar Angles

- Earth is divided into 369 deg of circular arc by longitudinal lines.
- Each 15 deg corresponds to 1/24 of a day that is 1 hour of time.
- **Universal Time Coordinate (UTC) or Greenwich Civil Time (GCT):** is the time along the zero longitude line passing through Greenwich England.
- The difference is being 4min/deg of longititud.
- **Local Civil Time (LCT):** is the time that is determined by the longitude of the observer.
- In some countries time is advanced one hour during summer leading to time saving and it is called Daylight Saving Time (DST)
- **Standard time:** the local civil time for a selected meridian near the center of the zone is called the standard time.
- **Local Standard Time (LST)= Local DST-1hr**
- **Local Solar Time (LST):** is the time calculated according to the position of sun and it is slightly different from civil time, which is precisely 24 hours according to 1- nonsymetry of earth's orbit 2- irregularity of earth rotational speed .
- LST= LCT+ (equation of time EOT)

Eastern standard time, EST 75 deg
Central standard time, CST 90 deg
Mountain standard time, MST 105 deg
Pacific standard time, PST 120 deg

In much of the United States clocks are advanced one hour during the late spring, summer, and early fall season, leading to *daylight savings time* (DST). Local standard time = Local DST − 1 hr.

Whereas civil time is based on days that are precisely 24 hours in length, solar time has slightly variable days because of the nonsymmetry of the earth's orbit, irregularities of the earth's rotational speed, and other factors. Time measured by the position of the sun is called *solar time*.

The local solar time (LST) can be calculated from the LCT with the help of a quantity called the *equation of time*: LST = LCT + (equation of time). The following relationship, developed from work by Spencer (2), may be used to determine the equation of time (EOT) in minutes:

$$EOT = 229.2\,(0.000075 + 0.001868\cos N - 0.032077\sin N - 0.014615\cos 2N - 0.04089\sin 2N) \tag{7-4}$$

where $N = (n-1)(360/365)$, and n is the day of the year, $1 \le n \le 365$. In this formulation, N is given in degrees. Values of the equation of time are given in Table 7-2 for the twenty-first day of each month (3).

The procedure for finding LST at a location with longitude L_L may be summarized as follows:

$$\text{If DST is in effect, Local Standard Time} = \text{Local DST} - 1\text{ hour} \tag{7-5}$$

$$\text{LST} = \text{Local Standard Time} - (L_L - L_S)(4\text{ min/deg W}) + EOT \tag{7-6}$$

EXAMPLE 7-1

Determine the LST corresponding to 11:00 A.M. Central Daylight Savings Time (CDST) on May 21 in Lincoln, NE (96.7 deg W longitude).

SOLUTION

It is first necessary to convert CDST to CST:

$$\text{CST} = \text{CDST} - 1\text{ hour} = 11{:}00 - 1 = 10{:}00\text{ A.M.}$$

From Table 7-2 the equation of time is 3.3 min. Then, using Eq. 7-6,

$$\text{LST} = 10{:}00 - (96.7 - 90)(4\text{ min/deg W}) + 0{:}03.3 = 9{:}37\text{ A.M.}$$

Table 7-2 Solar Data for Twenty-First Day of Each Month[a]

	Equation of Time, min	Declination, degrees	A, $\frac{\text{Btu}}{\text{hr-ft}^2}$	A, $\frac{\text{W}}{\text{m}^2}$	B, Dimensionless	C, Dimensionless
Jan	−11.2	−20.2	381.0	1202	0.141	0.103
Feb	−13.9	−10.8	376.2	1187	0.142	0.104
Mar	−7.5	0.0	368.9	1164	0.149	0.109
Apr	1.1	11.6	358.2	1130	0.164	0.120
May	3.3	20.0	350.6	1106	0.177	0.130
June	−1.4	23.45	346.1	1092	0.185	0.137
July	−6.2	20.6	346.4	1093	0.186	0.138
Aug	−2.4	12.3	350.9	1107	0.182	0.134
Sep	7.5	0.0	360.1	1136	0.165	0.121
Oct	15.4	−10.5	369.6	1166	0.152	0.111
Nov	13.8	−19.8	377.2	1190	0.142	0.106
Dec	1.6	−23.45	381.6	1204	0.141	0.103

[a]A, B, C, coefficients are based on research by Machler and Iqbal (6).

Source: Reprinted by permission from *ASHRAE Cooling and Heating Load Calculation Manual*, 2nd ed., 1992.

- The direction of the sun ray can be determined if knowing the following: 1- Location on the earth's surface 2-Time of the day 3-Day of the year
- The previously mentioned quantities can be describes by giving the latitude, the hour angle and the sun's declination
- Latitude (L) it describes the location of a certain point on earth and it is the angle between a line measured from the centre of the earth to the point and the projection of it on the equatorial plane.
- Hour angle (h): is the angle measured between the projection of any point on the equatorial plane and the projection on that plane of a line from the centre of the sun to the centre of the earth.

h is positive at noon time

h is negative at morning time

h is zero at local solar noon

- The declination angle (δ): is the angle between a line connecting the center of the sun and earth and the projection of that line on the equatorial plane

$$\delta = 0.3963723 - 22.9132745 \cos N + 4.0254304 \sin N - 0.3872050 \cos 2N + 0.05196728 \sin 2N - 0.1545267 \cos 3N + 0.08479777 \sin 3N \quad (7\text{-}7)$$

where $N = (n - 1)(360/365)$, and n is the day of the year, $1 \leq n \leq 365$. In this formulation, N is given in degrees. Table 7-2 shows typical values of the sun's declination for the twenty-first day of each month.

It is convenient in HVAC computations to define the sun's position in the sky in terms of the solar altitude β and the solar azimuth ϕ, which depend on the fundamental quantities l, h, and δ.

The *solar altitude angle* β is the angle between the sun's ray and the projection of that ray on a horizontal surface (Fig. 7-4). It is the angle of the sun above the horizon. It can be shown by analytic geometry that the following relationship is true:

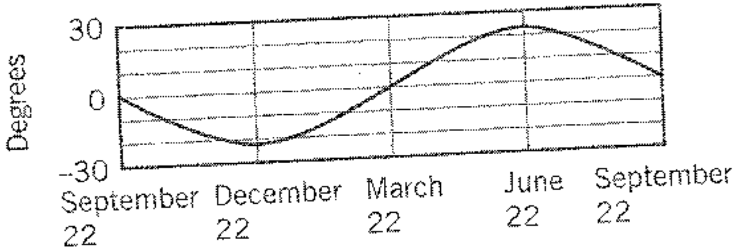

Figure 7-3 Variation of sun's declination.

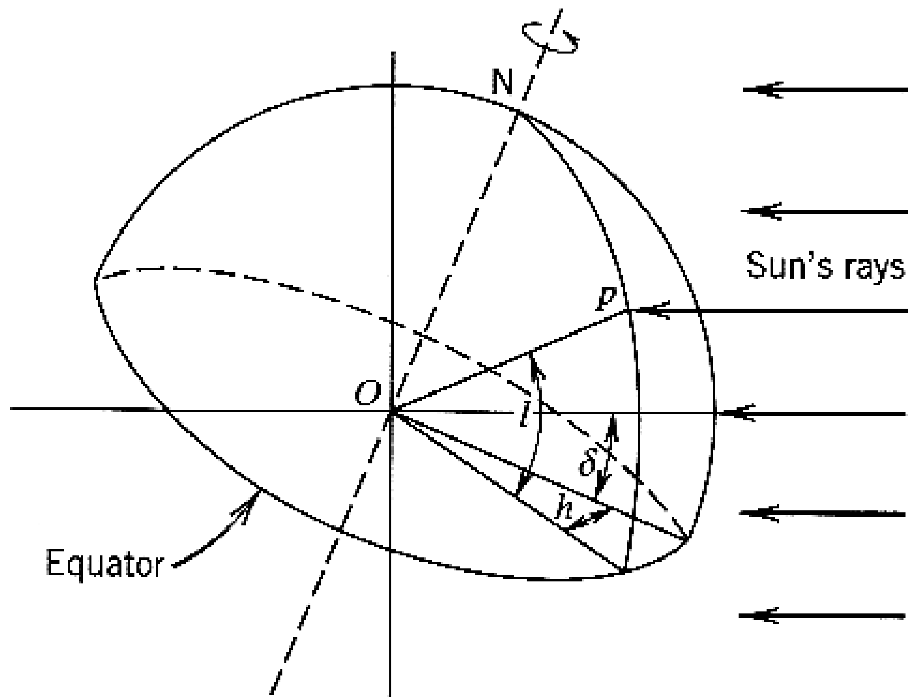

Figure 7-2. Latitude, hour angle, and sun's declination.

- Solar altitude angle (β): is the angle between the sun's ray and the projection of that ray on a horizontal surface and it represents the angle of sun above horizon.
- Sun's zenith angle (θ_z): is the angle between the sun's ray and a perpendicular to the horizontal plane at a given point.
- Solar azimuth angle (ϕ): is the angle in the horizontal plane measured in the clockwise direction, between north and the projection of the sun's ray on that plane.

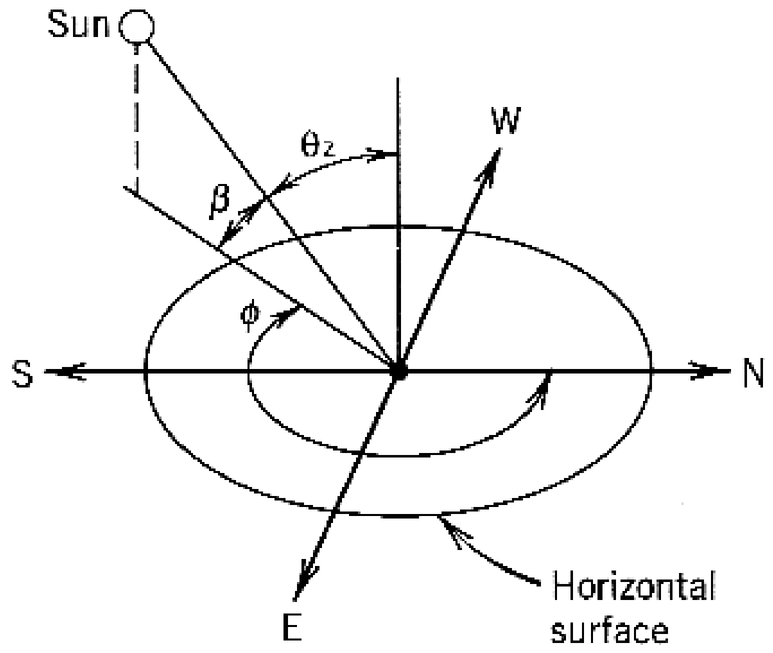

Figure 7-4 The solar altitude angle β and azimuth angle ϕ.

$$\sin \beta = \cos l \cos h \cos \delta + \sin l \sin \delta \tag{7-8}$$

The *sun's zenith angle* θ_Z is the angle between the sun's rays and a perpendicular to the horizontal plane at point P (Fig. 7-4). Obviously

$$\beta + \theta_Z = 90 \text{ degrees} \tag{7-9}$$

The daily maximum altitude (solar noon) of the sun at a given location can be shown to be

$$\beta_{\text{noon}} = 90 - |l - \delta| \text{ degrees} \tag{7-10}$$

where $|l - \delta|$ is the absolute value of $l - \delta$.

The *solar azimuth* angle ϕ is the angle in the horizontal plane measured, in the clockwise direction, between north and the projection of the sun's rays on that plane (Fig. 7-4). It might also be thought of as the facing direction of the sun. Again by analytic geometry it can be shown that

$$\cos \phi = \frac{\sin \delta \cos l - \cos \delta \sin l \cos h}{\cos \beta} \tag{7-11}$$

Note that, when calculating ϕ by taking the inverse of $\cos \phi$, it is necessary to check which quadrant ϕ is in.

For a vertical or tilted surface the angle measured in the horizontal plane between the projection of the sun's rays on that plane and a normal to the surface is called the *surface solar azimuth* γ. Figure 7-5 illustrates this quantity.

If ψ is the surface azimuth (facing direction) measured clockwise from north, then obviously

$$\gamma = |\phi - \psi| \tag{7-12}$$

The *angle of incidence* θ is the angle between the sun's rays and the normal to the surface, as shown in Fig. 7-5. The *tilt angle* α is the angle between the normal to the surface and the normal to the horizontal surface. Then a flat roof has a tilt angle of zero; a vertical wall has a tilt angle of 90 deg. It may be shown that

$$\cos \theta = \cos \beta \cos \gamma \sin \alpha + \sin \beta \cos \alpha \tag{7-13a}$$

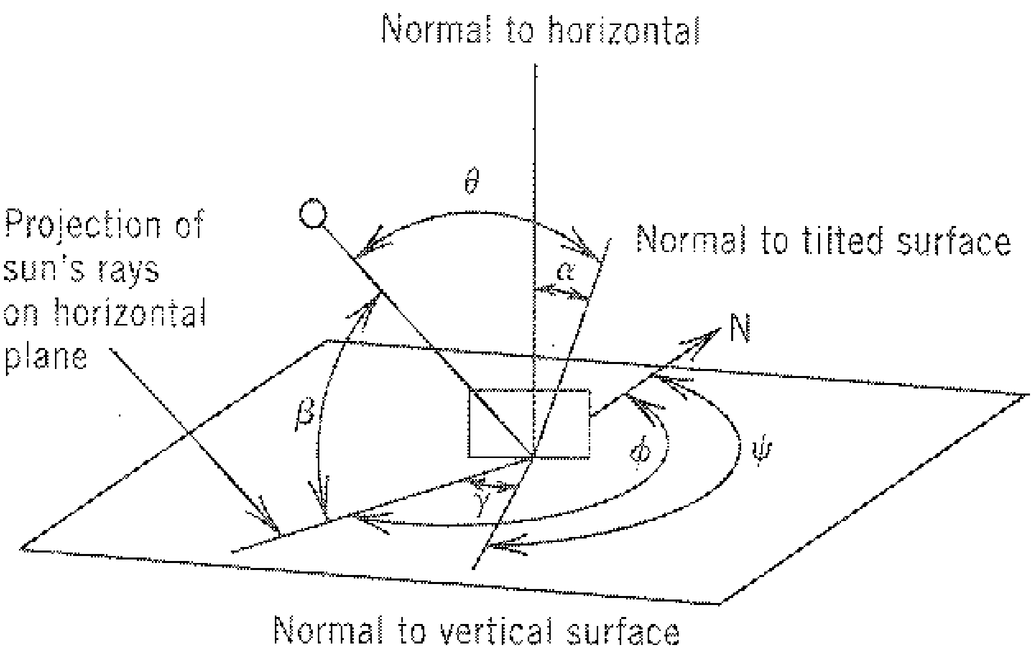

Figure 7-5 Surface solar azimuth γ, surface azimuth ψ, and angle of tilt α for an arbitrary tilted surface.

Solar Irradiation

- **Mean solar constant G_{sc}** : It is the rate of irradiation on the surface normal to the sun's rays beyond the earth's atmosphere and at the mean earth-sun distance and it is approximately = 433.4 Btu/(hr-ft^2) or 1367 W/m^2.
- **Direct or beam radiation G_{ND}**: it is that part of radiation that is not scattered or absorbed and reaches the earth's surface.
- **Diffuse radiation G_d:** it is the radiation that has been scattered or reemitted.
- **Reflected radiation G_R:** it is the radiation that is reflected from other surfaces.
- **Total irradiation $G_t = G_{ND} + G_d + G_R$**

ASHRAE Clear Sky Model

- The value of the solar irradiation at the surface of the earth on a clear day is given by the ASHRAE Clear Sky Model:

$$G_{ND} = \frac{A}{\exp(B/\sin\beta)} C_N \qquad (7\text{-}15)$$

where:

G_{ND} = normal direct irradiation, Btu/(hr-ft^2) or W/m^2
A = apparent solar irradiation at air mass equal to zero, Btu/(hr-ft^2) or W/m^2
B = atmospheric extinction coefficient
β = solar altitude
C_N = clearness number

On a surface of arbitrary orientation, the direct radiation, corrected for clearness, is:

$$G_D = G_{ND} \cos\theta \tag{7-16a}$$

where θ is the angle of incidence between the sun's rays and the normal to the surface. Note that if $\cos\theta$ is less than zero, there is no direct radiation incident on the surface—it is in the shade. If implementing this in a computer program, it might be more conveniently expressed as

$$G_D = G_{ND} \max(\cos\theta, 0) \tag{7-16b}$$

The diffuse irradiation on a horizontal surface is given by the use of the factor C from Table 7-2:

$$G_d = (C)(G_{ND}) \tag{7-17}$$

where C is obviously the ratio of diffuse irradiation on a horizontal surface to direct normal irradiation. The parameter C is assumed to be a constant for an average clear day for a particular month. In reality the diffuse radiation varies directionally (7) and

For nonhorizontal surfaces, the diffuse radiation $G_{d\theta}$ striking the surface may be calculated assuming the sky is isotropic (uniformly bright, excepting the sun) or anisotropic (brightness varies over the sky, e.g., around the sun and near the horizon). The ASHRAE model assumes an isotropic sky for all nonvertical surfaces. Vertical surfaces are treated as a special case with an anisotropic sky model.

First, to estimate the rate at which diffuse radiation $G_{d\theta}$ strikes a nonvertical surface on a clear day, the following equation is used:

$$G_{d\theta} = C\,G_{ND}F_{ws} \tag{7-18}$$

in which F_{ws} is the configuration factor or angle factor between the wall and the sky. The *configuration factor* is the fraction of the diffuse radiation leaving one surface that

would fall directly on another surface. This factor is sometimes referred to in the literature as the *angle factor* or *the view, shape, interception,* or *geometrical factor*. For diffuse radiation this factor is a function only of the geometry of the surface or surfaces to which it is related. Because the configuration factor is useful for any type of diffuse radiation, information obtained in illumination, radio, or nuclear engineering studies is often useful to engineers interested in thermal radiation.

A very important and useful characteristic of configuration factors is the reciprocity relationship:

$$A_1F_{12} = A_2F_{21} \tag{7-19}$$

Its usefulness is in determining configuration factors when the reciprocal factor is known or when the reciprocal factor is more easily obtained than the desired factor.

For example, the fraction of the diffuse radiation in the sky that strikes a given surface would be difficult to determine directly. The fraction of the energy that leaves the surface and "strikes" the sky directly, F_{ws}, however, can be easily determined from the geometry:

$$F_{ws} = \frac{1 + \cos\alpha}{2} \tag{7-20}$$

where α is the tilt angle of the surface from horizontal in degrees.

The rate at which diffuse radiation from the sky strikes a given surface of area A_w is, per unit area of surface,

$$\frac{\dot{q}}{A_w} = \frac{A_s G_d F_{sw}}{A_w}$$

By reciprocity

$$A_s F_{sw} = A_w F_{ws}$$

$$A_s F_{sw} = A_w F_{ws}$$

Therefore,

$$\frac{\dot{q}}{A_w} = G_d F_{ws}$$

Thus, although the computation involves the irradiation of the sky on the surface or wall, the configuration factor most convenient to use is F_{ws}, the one describing the fraction of the surface radiation that strikes the sky.

The use of the configuration factor assumes that diffuse radiation comes uniformly from the sky in all directions—an isotropic sky. This, of course, is an approximation. For vertical surfaces, the ASHRAE sky model takes into account the brighter circumsolar region of the sky. This is represented by the curve given in Fig. 7-8, which gives the ratio of diffuse sky radiation on a vertical surface to that incident on a horizontal surface on a clear day (7). The curve may be approximated (5) by

$$G_{dV}/G_{dH} = 0.55 + 0.437 \cos\theta + 0.313 \cos^2\theta \tag{7-21}$$

when $\cos\theta > -0.2$; otherwise, $G_{dV}/G_{dH} = 0.45$.

Then, for vertical surfaces, the diffuse sky radiation is given by:

$$G_{d\theta} = \frac{G_{dV}}{G_{dH}} C\, G_{ND} \tag{7-22}$$

In determining the total rate at which radiation strikes a nonhorizontal surface at any time, one must also consider the energy reflected from the ground or surroundings onto the surface. Assuming the ground and surroundings reflect diffusely, the reflected radiation incident on the surface is:

$$G_R = G_{tH} \rho_g F_{wg} \tag{7-23}$$

where:

G_R = rate at which energy is reflected onto the wall, Btu/(hr-ft^2) or W/m^2
G_{tH} = rate at which the total radiation (direct plus diffuse) strikes the horizontal surface or ground in front of the wall, Btu/(hr-ft^2) or W/m^2
ρ_g = reflectance of ground or horizontal surface
F_{wg} = configuration or angle factor from wall to ground, defined as the fraction of the radiation leaving the wall of interest that strikes the horizontal surface or ground directly

For a surface or wall at a tilt angle α to the horizontal,

$$F_{wg} = \frac{1 - \cos\alpha}{2} \tag{7-24}$$

To summarize, the total solar radiation incident on a nonvertical surface would be found by adding the individual components: direct (Eq. 7-16a), sky diffuse (Eq. 7-18), and reflected (Eq. 7-23):

$$G_t = G_D + G_d + G_R = \left[\max(\cos\theta, 0) + C\, F_{ws} + \rho_g F_{wg}(\sin\beta + C)\right] G_{ND} \tag{7-25}$$

Likewise, the total solar radiation incident on a vertical surface would be found by adding the individual components: direct (Eq. 7-16a), sky diffuse (Eq. 7-22), and reflected (Eq. 7-23):

$$G_t = G_D + G_d + G_R = \left[\max(\cos\theta, 0) + \frac{G_{dV}}{G_{dH}} C + \rho_g F_{wg}(\sin\beta + C)\right] G_{ND} \quad (7\text{-}26)$$

Heat gain through fenestrations

- Fenestration refers to any glazed aperture in a building envelope and it includes:

1-Glazing material, either glass or plastic

2-Framing, mullions, muntins and dividers

3- External shading devices

4-Internal shading devices

5- Integral (between-glass) shading systems

- Fenestration affect rates of heat transfer into and out of building and they are considered part as a source of air leakage and provide day lighting.

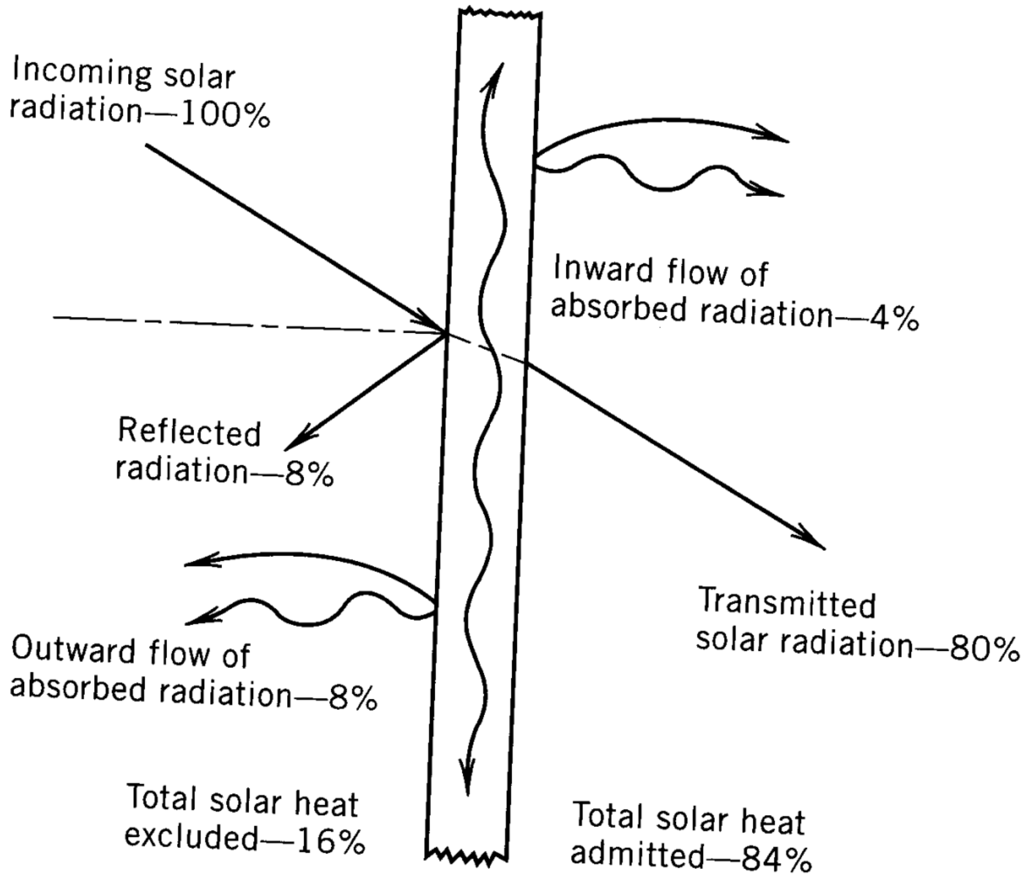

Figure 7-9 Distribution of solar radiation falling on clear plate glass.

$$\text{Total heat admission through glass} = \text{Radiation transmitted through glass} + \text{Inward flow of absorbed solar radiation} + \text{Conduction heat gain}$$

The first two quantities on the right are related to the amount of solar radiation falling on the glass, and the third quantity occurs whether or not the sun is shining. In winter the conduction heat flow may well be outward rather than inward. The total heat gain becomes

$$\text{Total heat gain} = \text{Solar heat gain} + \text{Conduction heat gain}$$

- **Sometimes the conduction heat gain and solar gain are approximated to be independent for a certain fenestration.**
- **In this case $q_{con} = U(t_o - t_i)A$**

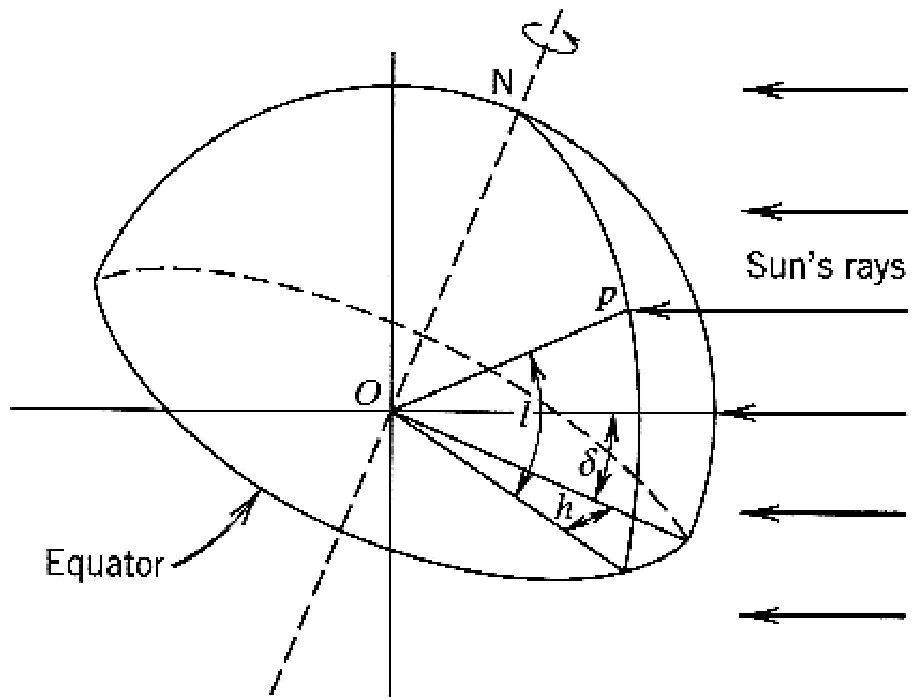

Figure 7-2. Latitude, hour angle, and sun's declination.

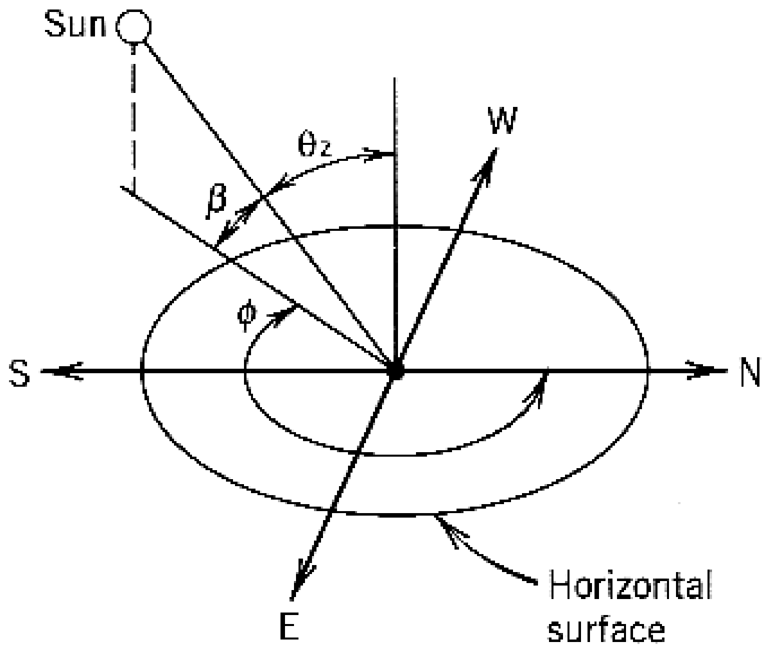

Figure 7-4 The solar altitude angle β and azimuth angle ϕ.

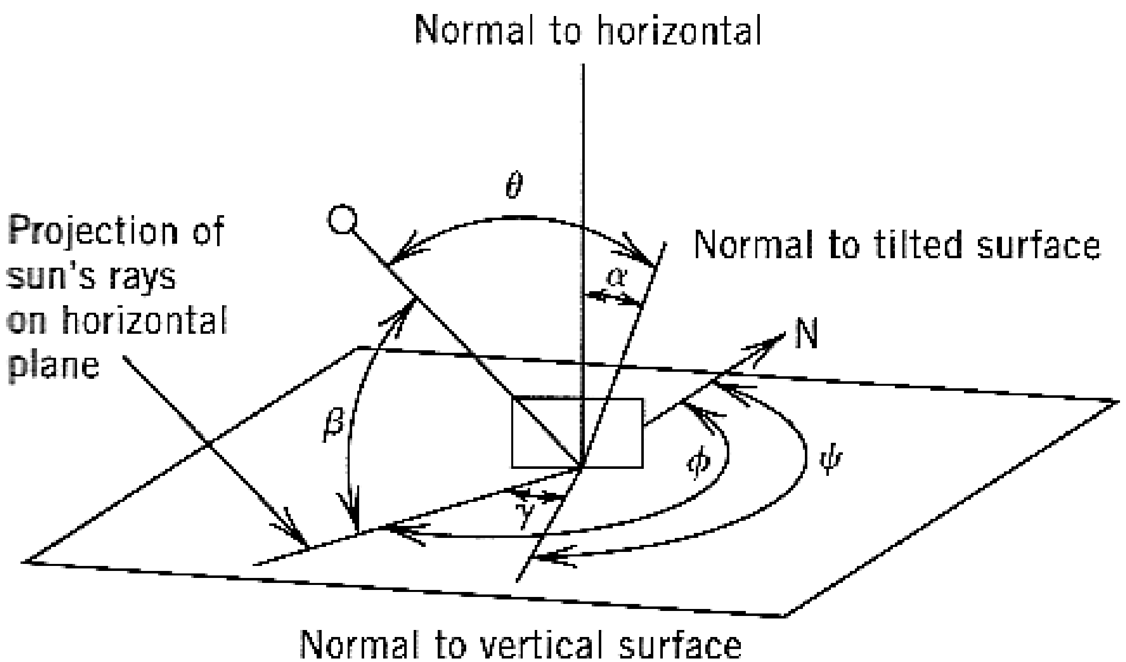

Figure 7-5 Surface solar azimuth γ, surface azimuth ψ, and angle of tilt α for an arbitrary tilted surface.

Solar Heat Gain Coefficients

- The heat gain through windows is complicated.
- A simplified method utilizes a spectrally-averaged solar heat gain coefficient SHGC, where the heat gain = G_i SHGC.
- The SHGC includes the directly transmitted portion, the inwardly flowing fraction of the absorbed portion.
- The data provided by the manufacturer mainly includes the normal radiation SHGC and the U factor.
- The SHGC approach does not treat the transmitted and absorbed components separately.
- Some procedures are used to calculate the heat gain:

1. The direct and diffuse solar radiation are calculated on an unshaded surface.
2. The effects of external shading on solar radiation incident on the window are determined.
3. The solar radiation transmitted and absorbed is analyzed for the window, assuming no internal shading.
4. If there is internal shading, its effects are then calculated.

External Shading

- A fenestration may be shaded by roof or overhangs or any other part, such as buildings, trees and etc.
- External shading reduces solar gain to a space, which can be up to 80%.
- To determine shading effect it is necessary to know the shaded area.
- It is assumed generally that shaded areas have no direct radiation, but the diffuse irradiation on the shaded area is the same s that on the sunlit area.
- The shading could have any shape and there are different models to calculate the shade area.

$$x = b \tan \gamma \quad (7\text{-}28)$$

$$y = b \tan \Omega \quad (7\text{-}29)$$

where:

$$\tan \Omega = \frac{\tan \beta}{\cos \gamma} \quad (7\text{-}30)$$

and *where:*

β = sun's altitude angle from Eq. 7-8
γ = wall solar azimuth angle = $|\phi - \psi|$ from Eq. 7-12
ϕ = solar azimuth from Eq. 7-11, measured clockwise from north
ψ = wall azimuth, measured clockwise from north

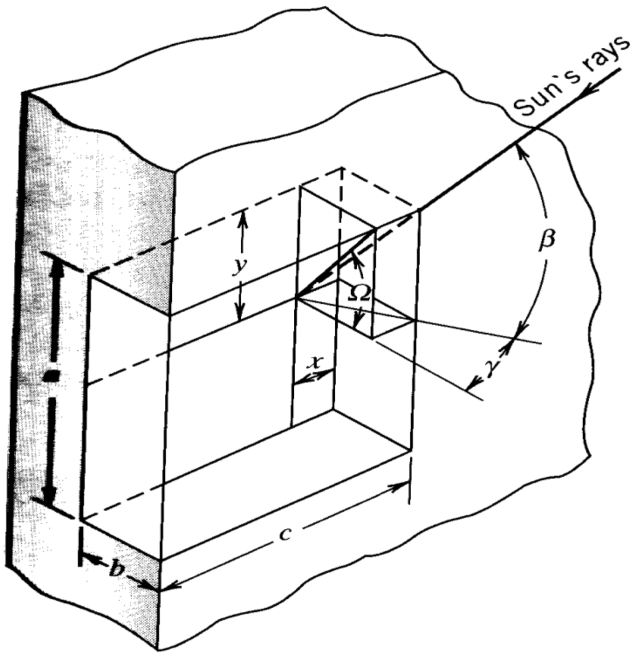

Figure 7-10 Shading of window set back from the plane of a building surface.

Transmission and Absorption of fenestration without Internal shading, simplified

In order to determine solar heat gain with the simplified procedure, it is assumed that, based on the procedures described above, the direct irradiance on the surface (G_D), the diffuse irradiance on the surface (G_d), the sunlit area of the glazing ($A_{sl,g}$), and the sunlit area of the frame ($A_{sl,f}$) are all known. In addition, the areas of the glazing (A_g) and frame (A_f) and the basic window properties must be known.

The solar heat gain coefficient of the frame ($SHGC_f$) may be estimated as

$$\mathrm{SHGC}_f = \alpha_f^s \left(\frac{U_f A_{frame}}{h_f A_{surf}} \right) \tag{7-31}$$

where A_{frame} is the projected area of the frame element, and A_{surf} is the actual surface area. α_f^s is the solar absorptivity of the exterior frame surface (see Table 7-1). U_f is the U-factor of the frame element (see Table 5-6); h_f is the overall exterior surface conductance (see Table 5-2). If other frame elements like dividers exist, they may be analyzed in the same way.

The solar heat gain coefficient of the glazing may be taken from Table 7-3 for a selection of sample windows. For additional windows, the reader should consult the *ASHRAE Handbook, Fundamentals Volume* (5) as well as the WINDOW software (12). There are actually two solar heat gain coefficients of interest, one for direct radiation at the actual incidence angle ($SHGC_{gD}$) and a second for diffuse radiation ($SHGC_{gd}$). $SHGC_{gD}$ may be determined from Table 7-3 by linear interpolation. Values of $SHGC_{gd}$ may be found in the column labeled "Diffuse."

Once the values of $SHGC_f$, $SHGC_{gD}$, and $SHGC_{gd}$ have been determined, the total solar heat gain of the window may be determined by applying direct radiation to the sunlit portion of the fenestration and direct and diffuse radiation to the entire fenestration:

$$\dot{q}_{SHG} = \left[SHGC_{gD}A_{sl,g} + SHGC_f A_{sl,f}\right]G_{D\theta} + \left[SHGC_{gd}A_g + SHGC_f A_f\right]G_{d\theta} \tag{7-32}$$

To compute the total heat gain through the window, the conduction heat gain must be added, which is estimated as

$$\dot{q}_{CHG} = U(t_o - t_i) \tag{7-33}$$

where U for the fenestration may be taken from Table 5-5, the *ASHRAE Handbook, Fundamentals Volume* (5), or the WINDOW 5.2 software (12); and $(t_o - t_i)$ is the outdoor–indoor temperature difference.

Transmission and absorption of fenestration without internal shading

- Absorbed solar radiation may flow into the space or back outside.
- The transmitted solar radiation depends on the angle of incidence and it is the highest when the angle in near zero.
- Transmittances are tabulated in table 7-3, as well as the transmittance for diffuse radiation is also given.
- Interpolation between angles is possible in table 7-3, which indicates linear relationships.
- An exact equation can also be used to determine the transmittance.

$$T_{D\theta} = \sum_{j=0}^{5} t_j[\cos\theta]^j \tag{7-34}$$

Once the direct transmittance has been determined, the transmitted solar radiation may be computed by summing the contributions of the direct radiation (only incident on the sunlit area of the glazing) and the diffuse radiation (assumed incident over the entire area of the glazing) as

$$\dot{q}_{TSHG,g} = T_{D\theta}G_{D\theta}A_{sl,g} + T_d G_{d\theta}A_g \tag{7-35}$$

where $\dot{q}_{TSHG,g}$ is the total transmitted solar radiation through the glazed area of the fenestration, $A_{sl,g}$ is the sunlit area of the glazing, and A_g is the area of the glazing.

The absorbed solar radiation also depends on the incidence angle, and layer-by-layer absorptances are also tabulated in Table 7-3. It should be noted that absorptances apply to the solar radiation incident on the outside of the window; for the second and third layers, the absorbed direct solar radiation in that layer would be calculated by multiplying the absorptance by $G_{D\theta}$. The total solar radiation absorbed by the K glazing layers is then given by

$$\dot{q}_{ASHG,g} = G_{D\theta}A_{sl,g}\sum_{k=1}^{K}\mathcal{A}^{f}_{k\,D\theta} + G_{d\theta}A_{g}\sum_{k=1}^{K}\mathcal{A}^{f}_{k\,d} \qquad (7\text{-}36)$$

where the absorptances for the kth layer, $\mathcal{A}^{f}_{k\,D\theta}$, are interpolated from Table 7-3. The superscript f specifies that the absorptances apply for solar radiation coming from the front or exterior of the window, not for reflected solar radiation coming from the back of the window.

It is then necessary to estimate the inward flowing fraction, N. A simple estimate may be made by considering the ratio of the conductances from the layer to the inside and outside. For the kth layer, the inward flowing fraction is then given by

$$N_k = \frac{U}{h_{o,k}} \qquad (7\text{-}37)$$

where U is the U-factor for the center-of-glazing and $h_{o,k}$ is the conductance between the exterior environment and the kth glazing layer. Then the inward flowing fraction for the entire window is given by

$$N = \frac{\left[G_{D\theta} \sum_{k=1}^{K} \mathcal{A}_{k\,D\theta}^{f} N_k + G_{d\theta} \sum_{k=1}^{K} \mathcal{A}_{k\,d}^{f} N_k \right]}{G_{D\theta} + G_{d\theta}} \tag{7-38}$$

In addition to the solar radiation absorbed by the glazing, a certain amount is also absorbed by the frame and conducted into the room. It may be estimated as

$$\dot{q}_{ASHG,f} = \left[G_{D\theta} A_{sl,f} + G_{d\theta} A_f \right] \alpha_f^s \left(\frac{U_f A_f}{h_f A_{surf}} \right) \tag{7-39}$$

where A_f is the projected area of the frame element, and A_{surf} is the actual surface area. α_f^s is the solar absorptivity of the exterior frame surface. U_f is the U-factor of the frame element, and h_f is the overall surface conductance. If other frame elements such as dividers exist, they may be analyzed in the same way.

Finally, the total absorbed solar radiation for the fenestration is

$$\dot{q}_{ASHG,gf} = N\,\dot{q}_{ASHG,g} + \dot{q}_{ASHG,f} \tag{7-40}$$